Phenomenology at low Q^2

Lecture I

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"Low Q^2 processes"

means such for which Q^2 constitutes a scale and this scale is too small, say $Q^2 \lesssim 1$ GeV^2, to apply a perturbative QCD.

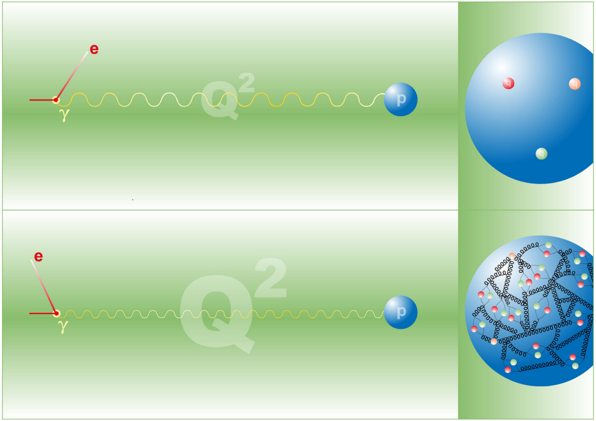
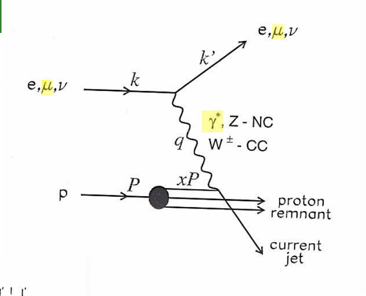


Figure from www.desy.de

Outline

Outline

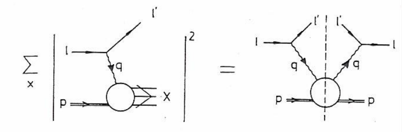
Definitions



Assume: one–γ^* approximation (Born approx.)
and LAB $\equiv$ target at rest.

LAB lepton energies: E and E'; M – proton mass;
Four-mom. transfer: $Q^2 = -q^2 \approx 4EE' sin^2\theta/2$
Bjorken scaling variable in LAB: $x = Q^2/(2M\nu)$
LAB energy transfer: $\nu = E - E'$;
Mass of produced hadronic syst.: $W^2 = M^2 + 2M\nu - Q^2$;
Frequent notation: $W^2 \equiv s$ (= $\gamma^* p$ cms energy^2).

Optical theorem:



Imaginary part of the forward Compton scattering amplitude of γ^* is defined by the tensor $W^{\mu\nu}$:

$$W^{\mu\nu}(p,q) = \frac{F_1(x,Q^2)}{M}\left(-g_{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + \frac{F_2(x,Q^2)}{M(pq)}\left(p^\mu - \frac{pq}{q^2}q^\mu\right)\left(p^\nu - \frac{pq}{q^2}q^\nu\right) \qquad (1)$$

$F_1(x,Q^2)$, $F_2(x,Q^2)$ – structure functions of the target, normalised to A; only two due to parity conservation. $W^{\mu\nu}$ satisfies the current conservation constraints:

$$q_\mu W^{\mu\nu} = 0, \qquad q_\nu W^{\mu\nu} = 0.$$

Definitions...cont'd

Rearrange eq.(1) to display potential kinematic singularities of $W^{\mu\nu}$ at Q^2=0:

$$W^{\mu\nu}(p,q) = -\frac{F_1}{M}g^{\mu\nu} + \frac{F_2}{M(pq)}p^\mu p^\nu + \left(\frac{F_1}{M} + \frac{F_2}{M}\frac{pq}{q^2}\right)\frac{q^\mu q^\nu}{q^2} - \frac{F_2}{M}\frac{p^\mu q^\nu + p^\nu q^\mu}{q^2}. \qquad (2)$$

These singularities cannot be real, and appear only as artifacts. To eliminate them we impose the following conditions at $Q^2 \rightarrow 0$:

$$F_2 = O(Q^2), \qquad \frac{F_1}{M} + \frac{F_2}{M}\frac{pq}{q^2} = O(Q^2). \qquad (3)$$

They must be fulfilled for arbitrary ν.

The electroproduction cross section is:

$$\frac{d^2\sigma(x,Q^2)}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1-y-\frac{Mxy}{2E}\right)\frac{F_2(x,Q^2)}{x} + \left(1-\frac{2m^2}{Q^2}\right)y^2F_1(x,Q^2)\right], \qquad (4)$$

where E denotes the energy of the incident lepton in the target rest frame, m is the lepton mass, $y=\nu/E$ and α is the electromagnetic coupling constant.

Definitions...cont'd

Instead of F_1 the function R is often used:

$$R(x,Q^2) = \frac{\sigma_L}{\sigma_T} = \frac{(1+4M^2x^2/Q^2)F_2}{2xF_1} - 1, \tag{5}$$

where σ_L and σ_T denote the cross sections for the longitudinally and transversally polarised virtual photons. Then the cross section:

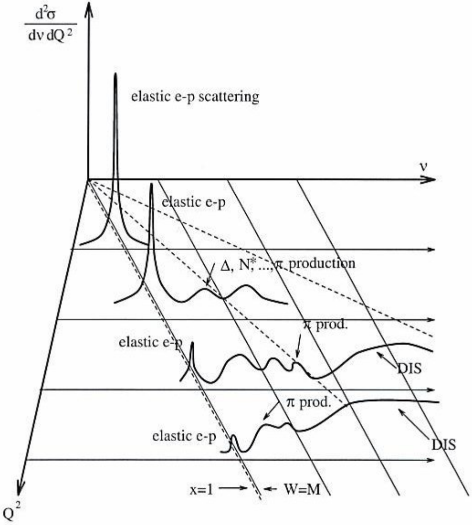
$$\frac{d^2\sigma(x,Q^2)}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\frac{F_2}{x}\left[1 - y - \frac{Mxy}{2E} + \left(1 - \frac{2m^2}{Q^2}\right)\frac{y^2(1+4M^2x^2/Q^2}{2(1+R)}\right]. \tag{6}$$

For the real photons ($Q^2 = 0$): $\sigma_L = 0 \rightarrow R = 0$.

The frequently used longitudinal structure function $F_L(x,Q^2)$ is defined:

$$F_L(x,Q^2) = \left(1 + \frac{4M^2x^2}{Q^2}\right)F_2 - 2xF_1. \tag{7}$$

At large Q^2, F_L is directly sensitive to the gluon distribution function. DIS region: $\nu, Q^2 \rightarrow \infty, x \rightarrow$ finite and O(1).



Radial, broken lines: x =const.
Parallel, continuous lines: W =const.

Low x – large parton (gluon) densities.
Low Q^2 – nonperturbative effects.

Outline

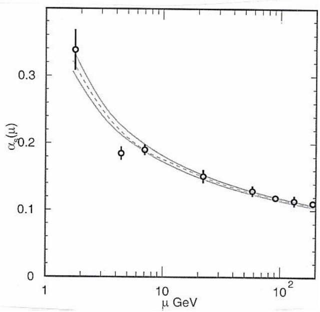
1. Definitions
2. Introduction
   - Why is the low Q^2 region important?
   - Acceptance of high energy electroproduction experiments
   - Ways to reach the low Q^2 (but not resonance) region
3. Basic concepts relevant to the small Q^2, small x region
   - F_2 in the Leading Ln (Q^2) approximation
   - Small x in DIS
   - Operator product expansion and higher twists
   - (Generalised) Vector Meson Dominance, (G)VMD
   - The Regge model
   - High-energy photoproduction
   - Diffraction
4. Physics domains
5. Summary

- It is great *par excellence*.
- Bulk of the data is soft.
- Continuity of physics: *nucleus* $\rightarrow$ *nucleons* $\rightarrow$ *quarks*, $\sqrt{Q^2} \cdot r \sim h$; photoproduction!
- Describing the fixed target data: low $x \longleftrightarrow$ low Q^2
- Analysing the DIS data: radiative corrections need knowledge of F_2 at $0 \leq Q^2 \leq Q^2_{meas}$.

However: at $Q^2 \rightarrow \Lambda^2_{QCD}$, $\alpha_s(Q^2) \rightarrow$"$\infty$" !

(Λ_{QCD} $\sim$0.2 GeV)

on the figure taken from PDG2006 $\mu \equiv \sqrt{Q^2}$



Outline

Acceptance of high energy electroproduction experiments

Kinematical domains for colliders and fixed target experiments

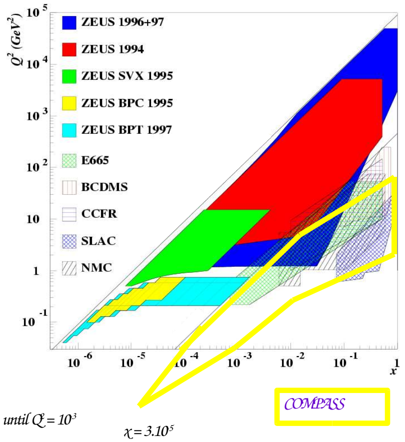
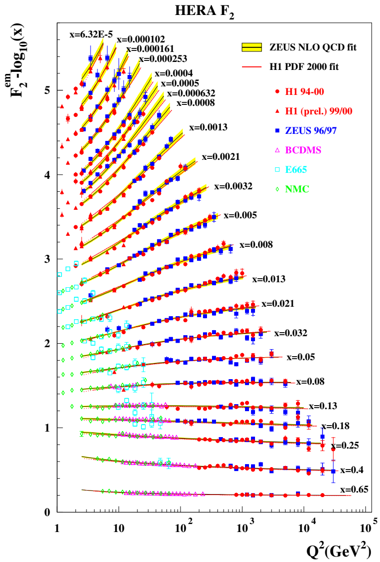


Figure from: N. D'Hose, Villars 2004

Experiments completed: E665 (FNAL; μ), BCDMS, NMC (CERN; μ), SLAC (e)
Ongoing experiments: ZEUS, H1 (HERA; ep; 30-820 GeV; $\sqrt{s}$ = 314 GeV),
COMPASS (CERN; μd; 160 GeV; $\sqrt{s}$ = 17 GeV)

Introduction...cont'd

Acceptance of high energy electroproduction experiments...cont'd



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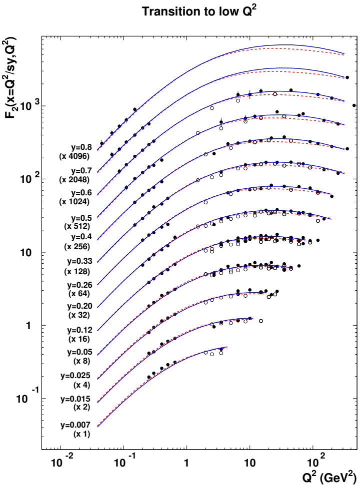
Introduction...cont'd

Acceptance of high energy electroproduction experiments...cont'd



$\Downarrow x$

Data: H1 and ZEUS;
curves (plotted for $x < 0.01$): the saturation model of Bartels, Golec-Biernat, Kowalski, Phys. Rev. D66 (2002) 014001.

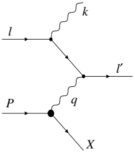
Outline

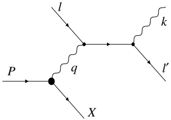
Introduction...cont'd

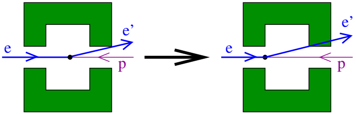
Ways to reach the low Q^2 (but not resonance) region

Low Q^2 means small beam scattering angles !

- Fixed target experiments (NMC, E665, SMC, COMPASS): low Q^2 demands event triggers with hadrons in the final state. Otherwise interaction vertex poorly defined.
- Colliders (HERA):
  - detector close to outgoing electron, BPC ($Q^2 \gtrsim .05$ GeV^2, lowest x)
  - shifted interaction vertex, SVTX ($Q^2 \sim 1$ GeV^2, low x)
  - Initial State Radiation, ISR ($Q^2 \sim 1$ GeV^2, low x)
  - QED Compton scattering, QEDC ($Q^2 \sim 1$ GeV^2, moderate x)





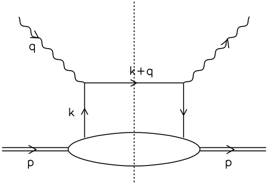


Outline

- In the leading ln(Q^2) approximation (LLQ^2) of QCD:

$$F_2(x, Q^2) = x \sum_i e_i^2 \left[q_i(x, Q^2) + \bar{q}_i(x, Q^2) \right] \tag{8}$$

where i labels the quark flavours and e_i are quark charges.



For large Q^2, x acquires the meaning of the fraction of the proton momentum carried by a struck quark. In this region q and $\bar{q}$ (or parton) distributions exhibit approximate Bjorken scaling mildly violated by the QCD logarithmic corrections. The evolution of these distributions with Q^2, i.e. $\partial q_i(x, Q^2)/\partial \ln(Q^2/\Lambda^2)$ and $\partial g(x, Q^2)/\partial \ln(Q^2/\Lambda^2)$, is described by the Altarelli–Parisi (or DGLAP) equations which (as well as eq.(8)) acquire corrections proportional to $\alpha_s(Q^2)$ in the next-to-leading ln(Q^2) approximation. In the LLQ^2 one keeps only those terms in the perturbative expansion which correspond to leading powers of lnQ^2, i.e. $\alpha_s^n \ln^n Q^2$.

Outline

Basic concepts relevant to the small Q^2, small x region

Small x in DIS

Total energy in the $\gamma^* p$ cms: $W = \sqrt{s}$. But

$$W^2 = M^2 + Q^2(1/x - 1) \tag{9}$$

then for $x \to 0$ (or $Q^2 \ll 2M\nu$ or $Q^2 \ll s$):

$$s = W^2 \to Q^2/x. \tag{10}$$

Now from the optical theorem applied to the $\sigma_{tot}(\gamma^* p \to \mathrm{hadrons})$, we introduce the amplitude, A, for the elastic Compton scattering $\gamma^* p \to \gamma^* p$ at the momentum transfer $\to 0$. Predictions for A are predictions for F_2 at low x.

At small x; sea nad gluons dominate valence quarks. Parton distributions generated by the LLQ^2 DGLAP in that limit $\Longrightarrow$ Double Logarithmic Approximation (DLA): retained are products of maximal powers of both large $\ln Q^2$ and $\ln 1/x$ which give:

$$xg \sim \exp(2\sqrt{\xi(Q^2)\ln(1/x)}) \tag{11}$$

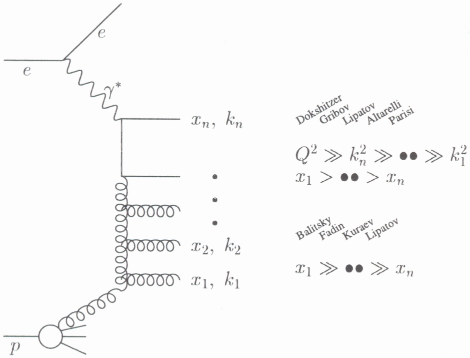
where

$$\xi(Q^2) = \int^{Q^2} \frac{dk^2}{k^2} \frac{3\alpha_s(k)}{\pi} \sim \ln\ln\left(\frac{Q^2}{\Lambda}\right) \tag{12}$$

(at low x and large Q^2).

But these are not all leading terms in parton densities at low x. Missing: leading powers of $\ln 1/x$ not accompanied by leading powers of $\ln Q^2$. LL$\frac{1}{x}$: leading powers of $\ln 1/x$ (arbitrary $\ln Q^2$) $\to$ equivalent of leading $\ln s$.

Diagrams which contribute to this approximation are ladder-like:



The BFKL equation which sums those diagrams leads to $xg \sim x^{1-\alpha_P^B}$ but the equation CCFM which treats both large $\ln 1/x$ and $\ln Q^2$ on equal footing gives a solution similar to that of conventional DGLAP.

OPE of electromagnetic currents, $W^{\mu\nu} \equiv \mathrm{Im} T^{\mu\nu} \propto \mathrm{Im} i \int d^4z < p|Tj_{em}^{\mu+}(z)j_{em}^{\nu}(0)|p >$, leads to:

$$F_2(x, Q^2) = \sum_{n=0}^{\infty} \frac{C_n(x, Q^2)}{(Q^2)^n} \tag{13}$$

$C_n(x, Q^2)$ depend weakly (i.e. logarithmically) on Q^2. Terms in (13) are: leading (n=0) and higher ($n \geq$1) twists; twist number: the leading one is 2 and higher ones correspond to consecutive even integers (4,6,...). Thus the rhs of eq.(8), with approximate Bjorken scaling of $q, \bar{q}$, corresponds to the LT contribution to F_2. For $Q^2 \lesssim$ few GeV^2 and lower, HT may become significant.

Contrary to common opinion, HTs are only corrections to the leading (approximately scaling) term (8) at large Q^2. They cannot correctly describe the low Q^2 region since (13) is divergent there; individual terms in (13) violate the constraint of (3). A correct way of describing that region: the formal expansion has to be summed beforehand, at large Q^2, and then continued to $Q^2 \sim$0. This is automatic in certain models, e.g.: the (Generalised) Vector Meson Dominance, (G)VMD.

Often in practise at moderate values of Q^2, the HT corrections are included as:

$$F_2(x, Q^2) = F_2^{LT}(x, Q^2)\left[1 + \frac{H(x)}{Q^2}\right] \tag{14}$$

$H(x)$ is determined from a fit to the data. Eq.(14) may not be justified theoretically, since C_n for $n \geq$1 in (13) evolve differently with Q^2 than the LT term.

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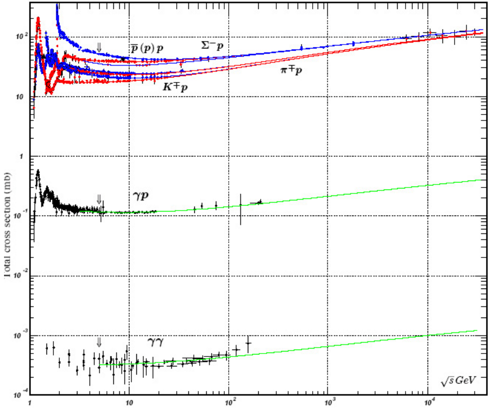
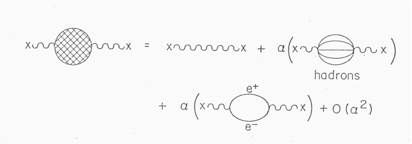
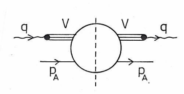
A consequence of an experimental fact that photon interactions are often similar to those of a hadron



Contributions to the self-energy of a physical photon:



The VMD-type interaction:



Hadrons in the γ fluctuation: either a pair of $q\bar{q}$ or a hadron of $J^P = 1^-$ (i.e. ρ, ω, Φ, J/Ψ,...). Observe that if E_γ is much larger than mass of the fluctuation, m, then the hadronic fluctuation traverses

$$d(E_\gamma, Q^2) \sim \frac{2E_\gamma}{Q^2 + m^2} \approx 80 \ \text{fm!!!} \ (\text{for } Q^2 = 0, E_\gamma = 100\,\text{GeV}, m^2 = 0.5\,\text{GeV}^2). \quad (15)$$

But a highly virtual γ^*, $Q^2 \to \infty$, may have no time to develop a structure before the interaction:

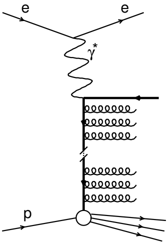
$$d(E_\gamma, Q^2) \sim \frac{2E_\gamma}{Q^2 + m^2} \to \frac{2E_\gamma}{Q^2} \to 0 \quad (16)$$

However the γ^* structure is visible! Observe that

$$\frac{2E_\gamma}{Q^2} = \frac{1}{Mx} \quad (17)$$

and if $x \ll 1$ then $d(E_\gamma, Q^2)$ may be very high independently of Q^2
(e.g. @ x=0.001, $d \sim 200$ fm! proton sea quarks outside proton ???)

Low x $\equiv$ large parton densities, due to QCD processes, e.g.:



Who is probing whom?? (A. Levy)

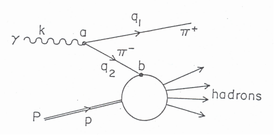
Solution: cross section is Lorentz invariant but time developement is not. (L. Frankfurt)
So γ^* and proton are probing each other and we are measuring the interation as a whole.

A consequence: @ low x, F_2^P and F_2^γ are related!

Basic concepts relevant to the small Q^2, small x region

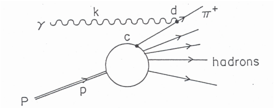
(Generalised) Vector Meson Dominance...cont'd

Two ways of γ interactions



dominant if $\nu \rightarrow \infty$ and target at rest

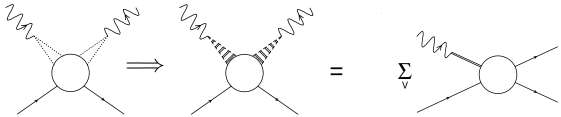
VMD defined

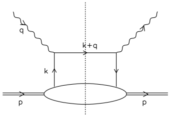


dominant in the ∞ target momentum system and finite ν

DIS defined

OR





$Q^2 \rightarrow 0$ (VMD)

$Q^2 \rightarrow \infty$ (DIS)

Transition between these regions ?

In the VMD model:

$$F_2\left[x=\frac{Q^2}{(s+Q^2-M^2)},Q^2\right]=\frac{Q^2}{4\pi}\sum_v\frac{M_v^4\sigma_v(s)}{\gamma_v^2(Q^2+M_v^2)^2}, \tag{18}$$

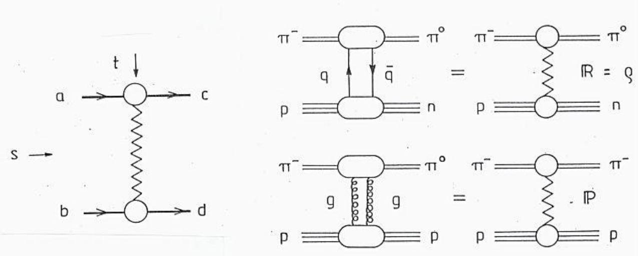
where $\sigma_v(s)$ are the VM–nucleon total cross sectins, M_v is the mass of the VM and γ_v^2 can be related to the leptonic width of the v:

$$\frac{\gamma_v^2}{\pi}=\frac{\alpha^2 M_v}{3\Gamma_{e^+e^-}}. \tag{19}$$

If the sum in eq.(18) is over a finite numver of VMs then F_2 vanishes as $1/Q^2$ at large Q^2 and thus it does not contain any LT term. Scaling can be introduced by including an infinite number of VMs in (18) $\longrightarrow$ GVMD.

Outline

Experimental fact: two-body scattering of hadrons dominated by small momentum transfers, t (or small scattering angles) is successfully described by the exchange of a particle with appropriate quantum numbers. The Regge model is used to parametrise high energy cross sections as functions of energy[2], W^2 (or s) and is based on a generalization of the particle exchange concept. Regge poles have quantum numbers: charge, isospin, strangeness,...(a "pole" from analytic structure of scattering amplitude).



A pole carrying the quantum numbers of the vacuum and describing diffractive scattering is called the pomeron (P); other poles – reggeon (R). No known particles can be associated with pomeron.

The Regge model...cont'd

Regge pole exchange $\equiv$ exchange of states of appropriate q.n., virtuality t and spin α. Relation $\alpha(t)$ – Regge trajectory; it interpolates between particles of different spins.

Energy behaviour of a two–body scattering amplitude due to a Regge pole exchange:

$$A(s,t) \sim s^{\alpha(t)};$$

and together with the optical theorem:

$$Im\ \ A(\text{elastic}, t=0) = s\sigma_{tot}$$

we get the following total cross section for $a + b \to \text{anything}$:

$$\sigma_{tot} \sim s^{\alpha(0)-1}$$

Here $\alpha(0)$ – trajectory intercept (universal, indep. of external particles or currents and dependent only on the quantum numbers of the poles exchanged in the crossed channel). More generally:

$$\sigma_t(W) = \sum_i \beta_i (W^2)^{\alpha_i(0)-1}$$

For the pomeron the expected intercept: $\alpha(0)$= 1; for other Regge poles $\alpha(0) \lesssim 0.5$.

Thus the Regge model gives the following parametrization of the F_2 at small x (and large Q^2):

$$F_2(x, Q^2) = \sum_i \tilde{\beta}_i x^{1-\alpha_i(0)}$$

The singlet part (sea quarks, gluons) is at small x controlled by pomeron exchange; the nonsinglet (valence quarks) – by the A_2 reggeon exchange ($\alpha(0) \sim 0.5$).

Mechanism of HEP interactions (and nature of pomeron) is more complicated than the exchange of simple Regge poles. So the pomeron – a mechanism responsible for diffraction. It should describe the increase of cross sections with increase of energy. At photoproduction the pomeron has an intercept about 1.08 (fit to the data).

Since the small x limit of DIS is the Regge limit then the Regge phenomenology acquires a new content within QCD. In the BFKL equation where $xg(x, Q^2) \sim x^{1-\alpha_P^B}$, $\alpha_P^B \sim 1.5 \longrightarrow$ "bare" pomeron ?!

Outline

Basic concepts relevant to the small Q^2, small x region

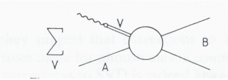
High-energy photoproduction ($Q^2 = 0$ limit)

$$\sigma_{\gamma p}(E_\gamma) = \lim_{Q^2 \to 0} 4\pi^2 \alpha \frac{F_2}{Q^2} \tag{20}$$

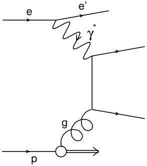
The limit should be taken at fixed $\nu = E_\gamma$. Conventional decomposition:

$$\sigma_{\gamma p} = \sigma_{VMD} + \sigma_{part}\,, \qquad \sigma_{part} = \sigma_{direct} + \sigma_{anomalous} \tag{21}$$

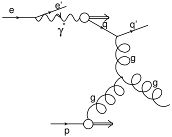
σ_{VMD} and σ_{part} are contributions from the VMD (light vector mesons only to avoid double counting) and partonic mechanisms; σ_{direct} – from γ^* interactions with partons in the hadron; $\sigma_{anomalous}$ – from partons in the γ^* interacting with partons in the hadron. Partonic mechanisms are hard $\rightarrow$ QCD. The VMD is both hard (partons from the VM) and soft. The anomalous term + the hard part of the VMD represent the pointlike interaction of the partonic constituents of γ^* ("resolved" γ^*).



VMD contribution



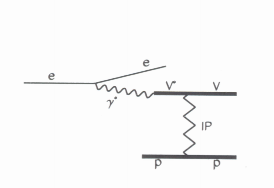
Direct γ



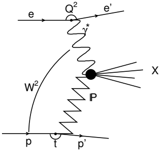
Resolved γ

The total $\sigma_{\gamma p}(W)$ rises with (high) energy and is well described by the soft pomeron contribution of $\alpha(0)$=1.08 (Donnachie and Landshoff). The rise reflects the small x behaviour of the parton distributions in a proton and a photon.

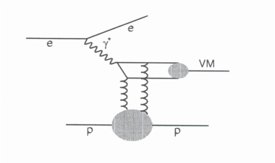
In classical optics: a redistribution of light incident on a small object. Same in the high energy photoproduction and for the hadronic beams; soft process. In 1993 a surprise: diffraction observed at HERA in DIS, i.e. in a hard process.



Photoproduction



DIS



DIS

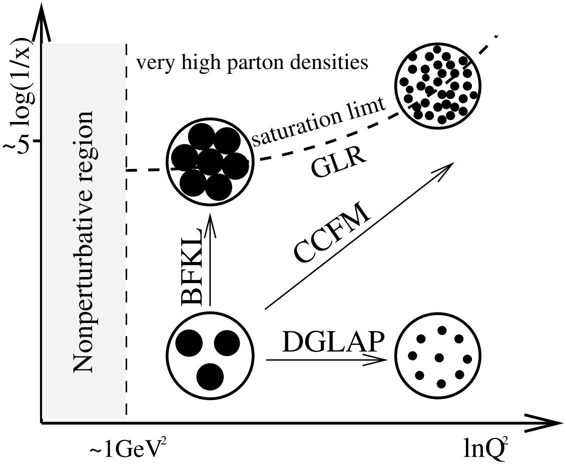
Diffraction characteristics: a beam (a photon, a hadron) diffracts into a system of same quantum numbers; the target remains intact or dissociates into a state of same quantum numbers; the exchange (pomeron) is colourless and carries the quantum number of the vacuum.
As a result: a large rapidity gap between the target and the system x.

Pomeron in the γ^* diffraction has an intercept $\alpha(0) >$1.08 (equal about 1.5; trajectory higher than for soft processes). Its parton structure studied (mostly gluons).

Outline

1. Definitions
2. Introduction
   - Why is the low Q^2 region important?
   - Acceptance of high energy electroproduction experiments
   - Ways to reach the low Q^2 (but not resonance) region
3. Basic concepts relevant to the small Q^2, small x region
   - F_2 in the Leading Ln (Q^2) approximation
   - Small x in DIS
   - Operator product expansion and higher twists
   - (Generalised) Vector Meson Dominance, (G)VMD
   - The Regge model
   - High-energy photoproduction
   - Diffraction
4. **Physics domains**
5. Summary

Physics domains



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Summary

- At high energies, low Q^2 region correlated with low x.
- Very important for understanding the transition from photoproduction to DIS; also for practical purposes.
- Several theoretical concepts relevant there.

- Outlook – next lectures
  - Spin-averaged electroproduction on nucleons and nuclei; data and models of F_2, R;
  - Spin-dependent electroproduction; data and models of g_1.